Rugby scores

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I. NUMBER OF TIMES A TEAM SCORED

Given the score of a rugby team, is it possible to know for instance how many times it scored, how many tries?

A. Parity

In rugby, penalty kicks and drop goals are worth 3 points each and tries are 5 points. After a try is scored a conversion is attempted, it is worth 2 points. Let n be the number of times a team scored and S the score of that team. Let p be the combined number of penalties and drop goals (3 points),[1] u the number of unconverted tries (5 points) and c the number of converted tries (7 points). All five numbers are in $\mathbb{N}$. S is known and all four others are unknown.

We have

$$n = p + u + c \tag{1}$$

and

$$S = 3p + 5u + 7c. \tag{2}$$

In passing, we can notice that the number of points scored kicking is

$$k = 3p + 2c.$$

Equation (2) can be rewritten

$$S = 5(p + u + c) + 2(c - p) = 5n + 2(c - p).$$

The second term on the right-hand side is clearly even, so S and n have the same parity:

$$n = S \mod 2. \tag{3}$$

By $a = b \mod c$, I mean that $|a - b|$ is a multiple of c (not necessarily between 0 and $c - 1$).

B. Minimum and maximum

Clearly

$$3n = 3(p + u + c) \leq 3p + 5u + 7c \leq 7(p + u + c) = 7n.$$

Since n is an integer,

$$\lceil S/7 \rceil \leq n \leq \lfloor S/3 \rfloor. \tag{4}$$

Here $\lfloor x \rfloor$ denotes the floor of x (its integral part) and $\lceil x \rceil$ its ceiling; for instance $\lfloor 2.4 \rfloor = 2$ and $\lceil 2.4 \rceil = 3$; for integers, $\lfloor 2 \rfloor = \lceil 2 \rceil = 2$.

C. Examples

If $S = 4$, we know that (i) n is even and (ii) $\lceil 4/7 \rceil \leq n \leq \lfloor 4/3 \rfloor$, i.e. $n = 1$. Since these two conditions are incompatible it is impossible to score 4 points.

If $S = 10$, (i) n is even and (ii) $\lceil 10/7 \rceil \leq n \leq \lfloor 10/3 \rfloor$, i.e. $2 \leq n \leq 3$; together it means that $n = 2$. Indeed the two ways to score 10 points are $5 + 5$ and $3 + 7$.

If $S = 16$, (i) n is even and (ii) $3 \leq n \leq 5$, i.e. $n = 4$. If a team scored 16 points, we can know with certainty that it scored exactly 4 times. If two converted tries were scored the total of 16 would be exceeded, so $c \in \{0, 1\}$. If $c = 0$ then $p + u = 4$ and $3p + 5u = 16$, which yields $p = u = 2$. If $c = 1$ then $p + u = 3$ and $3p + 5u = 9$, so $p = 3$ and $u = 0$. The only two ways of scoring 16 points are: two unconverted tries and two penalties, or one converted try and three penalties (obviously not necessarily in this order). The kicker scored either 6 or 11 points (assuming he did not score a try).

If $S = 15$ or $S = 17$, n is odd and $3 \leq n \leq 5$, i.e. $n \in \{3, 5\}$. $S = 15$ and $S = 17$ are thus more similar to each other than they are to $S = 16$.[2]

D. Recursion

How to score 17 points? Score (i) 10 points plus a converted try or (ii) 12 points plus an unconverted try or (iii) 14 points plus a penalty. This is clearly recursive: if one knows how to get all scores up to 16 points, one can easily find out how to score 17 points. $10 = 3 + 7$ or 2×5, $12 = 4 \times 3$ or $5 + 7$ and $14 = 2 \times 7$ or $3 \times 3 + 5$. Thus $17 = 3 + 2 \times 7$ or $2 \times 5 + 7$ or $4 \times 3 + 5$ or $2 \times 5 + 7$ or $3 + 2 \times 7$ or $4 \times 3 + 5$. Removing the redundant ones, there are just three possibilities: $3 + 2 \times 7$, $2 \times 5 + 7$ and $4 \times 3 + 5$. As expected, for all of them, $n \in \{3, 5\}$. We can see that there can be 0, 1 or 4 penalties, but not 2 or 3 (whereas for 16 points there could only be 2 or 3 penalties).

E. Increasing *S* by 21

Figure 1 shows how many possible values exist for n as a function of S. Whenever S increases by 21, the number of possibilities for n increases by 2 (leading to a slope of 2/21 for the linear fit). For instance, for $S = 0$ there is a single possibility ($n = 0$), whereas for $S = 21$ there are 3 possible values, $n \in \{3, 5, 7\}$. $S = 1$ and $S = 2$ are impossible, and for $S = 22$ or 23 there are 2 possible values of n.

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[1] Since from the point of view of scores penalties and drop goals are indistinguishable (and the latter are uncommon), 'penalty' will henceforth be used as a synonym for 'scoring 3 points'.

[2] At this point this may not come as a surprise given the role played by parity, but had I told you this ten minutes ago, you would have been sceptical.

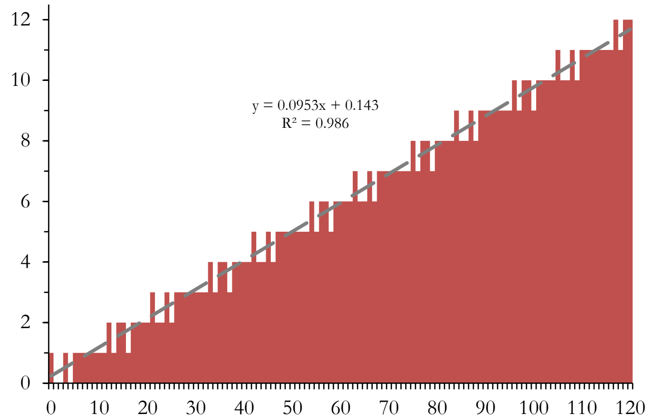

FIG. 1: The number of possibilities for n as a function of S.

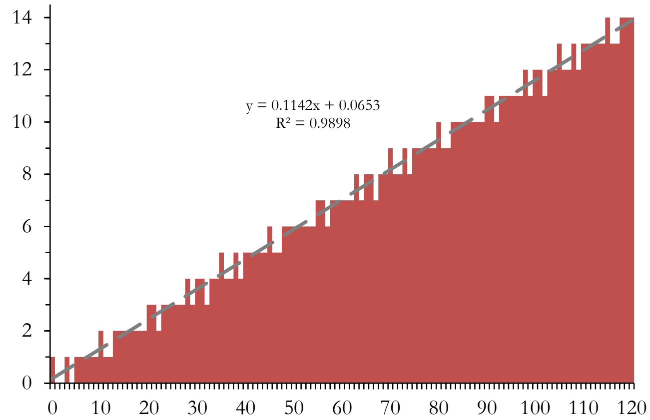

FIG. 2: Number of possible values for $c-u$ as a function of S.

If n (resp. n') is the number of times a team scored for a total of S (resp. $S+21$), from Eq. (4),

$$\lceil S/7 \rceil + 3 \leq n' \leq \lfloor S/3 \rfloor + 7.$$

If n could take values (of a certain parity) in $[n_1, n_2]$ then n' can take values (of the opposite parity) in $[n_1+3, n_2+7]$, i.e. it can take two more values.

II. THE TOTAL NUMBER OF TRIES

With championships where tries are used to calculate a bonus, the number of tries may be known (and displayed live on TV) along with the score.

A. Properties of the total number of tries T

We define the total number of tries

$$T = u + c. \tag{5}$$

Clearly

$$3p + 5u + 7c \geq 5(u+c) = 5T.$$

Since T is an integer,

$$T \leq \lfloor S/5 \rfloor. \tag{6}$$

Plainly, if S is not a multiple of 3, T cannot be zero. In that case, $T \geq 1$.

B. The difference between converted and unconverted tries, $c - u$

S can be rewritten as $S = 3(n+T) + (c-u)$. This yields right away $c - u = S \mod 3$.

- If S is a multiple of 3, so is $c-u$. (Most of the time, this will mean $u = c$.)
- If S is a multiple of 3 plus 1 then $c - u = 1 \mod 3$. In particular there must be at least one converted try or two unconverted tries.
- If S is a multiple of 3 minus 1 then $c-u = -1 \mod 3$ and there must be at least one unconverted try or two converted tries.

Since $c \geq 0$ and $u \geq 0$,

$$-\lceil S/5 \rceil \leq -u \leq c-u \leq c \leq \lfloor S/7 \rfloor.$$

Similar to Sec. I E, if S is increased by 35,

$$-7 - \lceil S/5 \rceil \leq c' - u' \leq 5 + \lfloor S/7 \rfloor.$$

This increases the range for $c-u$ by 12. But since $c-u = S \mod 3$, only a third of this is accessible. Finally, if S is increased by 35, the range for $c-u$ increases by 4. This gives a slope of 4/35 for the linear fit in Fig. 2.

C. Calculating n if the number of tries is known

We have $T = 2u + (c-u)$, which shows that $c-u$ has the same parity as T. The maximum possible value of $c-u$ is T (if $c = T$ and $u = 0$) and the minimum is $-T$.

Since $c-u = T \mod 2$ and $c-u = S \mod 3$, we have

$$c - u = 6\mu + \Delta. \tag{7}$$

The integer Δ is known from S and T: $\Delta = T \mod 2$ and $\Delta = S \mod 3$. The integer μ is such that $-T-\Delta \leq 6\mu \leq T - \Delta$. In particular, if T is small then $\mu = 0$.

S then becomes $S = 3(n+T+2\mu)+\Delta$, so that

$$n = \frac{S-\Delta}{3} - T - 2\mu, \tag{8}$$

where S, T and Δ are known.

D. Example

Let us reuse the example of $S = 17$.

- Since S is not a multiple of 3, T cannot be zero.
- If $T = 1$ then $\mu = 0$, $\Delta = -1$ and $n = [17-(-1)]/3 - 1 - 0 = 5$: the only try is not converted and there are 4 penalties.
- If $T = 2$ then $\mu = 0$, $\Delta = 2$ and $n = (17-2)/3 - 2 - 0 = 3$: both tries are converted and there is 1 penalty.
- If $T = 3$ then $\mu = 0$, $\Delta = -1$ and $n = [17-(-1)]/3 - 3 - 0 = 3$: there is one more unconverted try than converted tries, and no penalties.
- $T \geq 4$ would mean $S \geq 20$.

We recover the three possibilities found earlier (but no recursion was needed this time).